We seek to prove Euler's identity : $e^{j\phi} = \cos\phi + j\sin\phi$

The Taylor Series is written :

$$f(x) = \frac{f(a)}{0!} + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

When $a = 0$, *we obtain the McLaurin Series* :

$$f(x) = \frac{f(0)}{0!} + \frac{f'(0)(x)}{1!} + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

We use the following derivatives : $\frac{d}{dx}e^x = e^x; \frac{d}{dx}\sin x = \cos x; \frac{d}{dx}\cos x = -\sin x$

Then by the McLaurin Series, $\sin x$ *and* $\cos x$ *are written as follows* :

$$\sin x = \sin 0 + \frac{\cos 0}{1!}x - \frac{\sin 0}{2!}x^2 - \frac{\cos 0}{3!}x^3 + \frac{\sin 0}{4!}x^4 + \frac{\cos 0}{5!}x^5 + \cdots = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos x = \cos 0 - \frac{\sin 0}{1!}x - \frac{\cos 0}{2!}x^2 + \frac{\sin 0}{3!}x^3 + \frac{\cos 0}{4!}x^4 + \cdots = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} + \cdots$$

$$\cos\phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} + \cdots$$

Also, by the McLaurin Series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

Re*call that* $j^0 = 1;\ j^1 = j;\ j^2 = -1;\ j^3 = -j;\ j^4 = 1;\ j^5 = j;$

thus with $x = j\phi$

$$e^{j\phi} = 1 + \frac{j\phi}{1!} + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \frac{(j\phi)^4}{4!} + \frac{(j\phi^5)}{5!} + \cdots$$

$$e^{j\phi} = 1 + j\phi - \frac{\phi^2}{2!} - \frac{j\phi^3}{3!} + \frac{\phi^4}{4!} + \frac{\phi^5}{5!} + \cdots$$

$$e^{j\phi} = (1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} + \cdots) + j(\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} + \cdots)$$

$$e^{j\phi} = \cos\phi + j\sin\phi$$

Q.E.D.